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ON THE ASYMPTOTIC LAYERING OF TURBULENT FLOWS

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The asymptotic structure of a turbulent boundary layer on a plate with a boundary-layer distributed suction, consisting of a suction zone, a viscous zone, a buffer zone, a velocity-defect zone, a Corrsin superlayer, and a irrotational-flow zone, has been determined. The analysis was carried out within the framework of the Reynolds equations with the use of the combined method of different scales and joined of asymptotic expansions. The Corrsin superlayer was interpreted as a discontinuity of turbulent stresses.

Introduction. In accordance with the synergetic laws, the structure of a dynamic system becomes complicated with increase in the number of determining parameters [1]. In the incompressible-fluid dynamics, the complication of the structure of a flow is related to its natural stratification — the formation of asymptotic layers. For example, the flow in a laminar boundary layer is unilaminate [2], the flow in an intensive laminar column-shaped vortex is two-layer [2], the flow in the neighborhood of the separation point of a laminar boundary layer (free interaction) is three-layer [3], the flow in a turbulent boundary layer is also three-layer [4, 5], and the flow in the zone of development of the disturbances of a supersonic flow is five-layer [6]. Thus, the *triad principle* [7, 8] in the fluid dynamics is not universal.

The semi-empirical and asymptotic definitions of a layer should be distinguished. In the semiempirical or empirical theory, a quantitative interval (layer), in which experimentally-discovered laws of wall, velocity defect, and wake act [9], is introduced. In the asymptotic theory [2–6], an internal variable related to the scale of the layer and changing usually from 0 to ∞ is introduced, the solutions in the neighboring layers join with each other and, within each layer, simplified equations following, with an *O*-evaluating accuracy, from the fluid-dynamics complete equations for the balance of the inertial, viscous, and turbulent forces are true. In the simplified equations, only certain combinations of these forces are retained and there exist the following capabilities: all the three types of forces are retained, they are involved in pairs (inertial and viscous forces, inertial and turbulent forces, viscous and turbulent forces) or individually (Euler equations, Stokes equations, the equality of turbulence forces to zero [10]).

V. I. Ponomarev has developed, with the use of the method of joining asymptotic expansions, a three-layer model of a turbulent boundary layer on a plate, consisting of a near-wall zone, a buffer zone, and a velocity-defect zone [4]. Thus, the asymptotic theory of turbulent boundary layer has been devised. Empirical theories were proposed in earlier works too [11, 12]. Even though the author used, for obtaining concrete results, the above-mentioned empirical laws of wall and velocity defect, the analysis carried out by him was fairly strict.

The boundary between the laminar and turbulent zones was experimentally investigated for the first time in [13]; it is called the *Corrsin superlayer* [9, 14]. It can be a boundary of both the extended turbulent zone (a jet, a boundary layer, a wake, a column-shaped vortex) and a turbulent spot in an intermittent flow. If the turbulization of a flow proceeds so rapidly that this boundary can be considered as a discontinuity, to the contrary, the relaminarization of the flow proceeds along the coordinate, and the corresponding boundary can hardly be considered as a discontinuity. The Corrsin superlayer dividing the turbulent and laminar zones should be not only in the wake of the jet or in the boundary layer, but also in the jet and in the mixing layer.

The objective of the present work is to theoretically investigate the properties of the experimentally-discovered Corrsin superlayer. Two standpoints on this layer exist. If the indicated superlayer is considered as a discontinuity in the corresponding scale, in it the conditions following from the first principles of continuum mechanics should be fulfilled. If this layer is considered as a superlayer having a thickness, the adapted Reynolds equations should be vapid in it.

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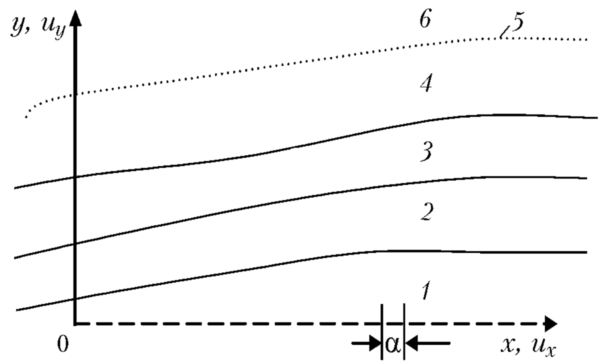


Fig. 1. Asymptotic structure of a flow: 1) suction zone; 2) viscous zone; 3) buffer zone; 4) velocity-defect zone; 6) irrotational-flow zone; the dashed line is the surface of a body with transverse slots; the dotted line is the Corrsin superlayer (zone 5).

Formulation of the Problem. A plane stationary flow of an incompressible Newtonian fluid propagating over a penetrable plane (Fig. 1) will be considered. As the base unit parameters, we will use the length of the plate, the velocity of an undisturbed flow, and the density of the fluid. In the Cartesian coordinate system x, y, in which the axis x is directed along the plate, the Reynolds equations have the form

$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = \nu \frac{\partial^2 u_x}{\partial y^2} - \frac{\partial \tau_{xy}}{\partial y}, \tag{1}$$

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0. \tag{2}$$

Expression (1) does not involve members that are small in the extended-turbulent-zone approximation being considered. Within the limits of this approach, the rate of suction is assumed to be small: $u_y(x, 0) = \beta v_0(x)$, $\beta \ll 1$. Let us assume that the suction is realized through the slots in the plane, arranged in a regular order with a period $\alpha = B(x)\nu^{n/2}$, where $B = O(1)$, $n > 0$. The penetrability of the slots is assumed to be arbitrary, equal to unity in order of magnitude.

As applied to the suction of fluid through the porous walls, in the fluid-flow model being considered, the stochastic disposition of pores and their shape are disregarded, which, however, is warranted because the Darcy model, used usually for calculating the filtration of flows, is impractical in the case of eddying-flow suction, i.e., in the case of boundary-layer suction.

In each asymptotic subregion, the internal coordinate $y_k = y/\delta_k$, where δ_k is a characteristic thickness of the kth subregion (k = 1–6), is introduced. A turbulent stress will be defined as $\tau_{xy} = \varepsilon_k \sigma_{xy}$, where $\varepsilon_k \ll 1$ and $\sigma_{xy} = O(1)$.

It is seen from the formulation of the problem that a priori there is an unknown function τ_{xy} and three parameters prescribed a priori: ν, α (or n instead of α), and β. In the asymptotic boundary-layer theory without suction, the logarithmic laws of wall and of velocity defect that conclusively agree with experimental results are additionally used. For the problem on the boundary-layer suction such laws were not established. Therefore, the analysis will be less concrete. As for the determining parameters, the solution becomes boundless in the case where their number is fairly large (≥3). In this case, the circumstance that the parameter α is of importance only in the suction zone, where β does not appear in the explicit form, alleviates the problem.

Near-Wall Viscous Region. The zonal structure of the turbulent boundary layer is determined by the opposite action of the friction and suction processes. First we consider the viscous near-wall zone 2 (k = 2). In accordance with the theory [4], at $\beta \equiv 0$ the vertical velocity in this region $u_y = O(\nu)$. Therefore, the suction begins to substantially influence the boundary layer at $\beta = \nu$. If $\beta \ll \nu$, the suction is only a linear addition to the flow without suction; otherwise, the vertical velocity can be represented as $u_y \approx \beta v_2(x, y_2)$. From the continuity equation (2), we find the ordinal number of the longitudinal velocity component $u_x \approx (\beta/\delta_2)u_2(x, y_2)$. In this zone, the inertial terms are small and the viscous and turbulent stresses involved on the right side of Eq. (1) are balanced. From this condition the thickness of the viscous layer $\delta_2 = (\nu\beta/\varepsilon_2)^{1/2}$ is determined. Upon integration of Eq. (1) with respect to y_2, we obtain the known law on the constancy of the friction stress across the layer

$$\frac{\partial u_2}{\partial y_2} - \sigma_{xy}(x, y_2) = A_2(x) . \tag{3}$$

The velocity of the flow increases with increase in β. It may be suggested that, at any critical value of $\beta = \beta_*$, along with terms responsible for the appearance of the viscosity and turbulence, the inertial terms entering in the left side of (1) play a dominant role in zone 2.

Suction Zone. In the suction zone 1, the flow is quasi-periodic along the longitudinal coordinate x. Since the period is small, for description of this flow we will use the difference-scale method [15] and, in doing so, introduce a rapid variable x_1 in accordance with the formula $dx_1/dx = 1/\alpha$.

The Navier–Stokes operator becomes nondegenerate when the transverse coordinate y is increased by α times: $y = \alpha y_1$. The slow variable x is retained; the solution remains dependent on this coordinate. Thus, instead of the two independent variables x and y, we will have three variables (x, x_1, and y). The dependence of x_1 on α is periodic, and the dependence of x_1 on x is nonperiodic. The fluid motion in the suction zone represents a flow around both sides of the infinite system of slots.

Let us estimate the orders of the quantities being considered. From (3) we have

$$\frac{\partial u_2}{\partial y_2}(x, 0) = A_2 - \sigma_{xy}(x, 0) .$$

Consequently, at $y_2 \to 0$

$$u_x \approx \frac{\beta}{\delta_2}\left[A_2 - \sigma_{xy}(x, 0)\right] y_2 = \frac{\alpha\beta}{\delta_2^2}\left[A_2 - \sigma_{xy}(x, 0)\right] y_1 .$$

The vorticity at the bottom of the viscous zone is constant. From the condition of join of the solution in zone 1 with the solution in zone 2 it follows that the vorticity in the suction zone at $y \to \infty$ is also constant and the velocity expansion has the form

$$u_x \approx \frac{\alpha\beta}{\delta_2^2} u_1(x, y_1) . \tag{4}$$

Moreover, at $\sigma_{xy}(x, 0) \neq 0$

$$\varepsilon_1 = \varepsilon_2 = \frac{\beta\nu}{\delta_2^2} . \tag{5}$$

The velocity is limited if the inequality $\beta << \delta_2^2/\alpha$ or the inequality $\beta = \delta_2^2/\alpha$ are fulfilled.

The Reynolds number in the suction zone is equal to

$$\mathrm{Re}_1 = \frac{\alpha^2\beta}{\nu\delta_2^2} . \tag{6}$$

There are three possibilities.

1. If $\alpha^2\beta = \nu\delta_2^2$, $R_1 = O(1)$ and the complete Reynolds equations are true since $\varepsilon_1 = (\nu/\alpha)^2$.

2. In the case where $\alpha^2\beta >> \nu\delta_2^2$, the viscosity forces are negligibly small. Using (4) and (5), we will compare, by order of magnitude, the inertial and turbulent terms in Eq. (1):

$$u_x \frac{\partial u_x}{\partial x} = O\left(\frac{u_x^2}{\alpha}\right) = O\left(\frac{\alpha\beta^2}{\delta_2^4}\right), \quad \frac{\partial \tau_{xy}}{\partial y} = O\left(\frac{\varepsilon_1}{\alpha}\right) = O\left(\frac{\beta}{\alpha}\frac{\nu}{\delta_2^2}\right).$$

Consequently, in the case being considered,

$$u_x \frac{\partial u_x}{\partial x} >> \frac{\partial \tau_{xy}}{\partial y}.$$

The flow in the suction zone is laminar. This conclusion is naturally plausible at σ_{xy} $(x, y_2 = 0) = 0$. In this case, the Euler equations are true.

3. Finally, in the case where $\alpha^2\beta << \nu\delta_2^2$, the inertial terms should be neglected. In Eq. (1), the viscous and turbulent terms are of the same order and are equal to $\beta\delta^2\delta_2^{-2}$. By analogy with (3) we have

$$\frac{\partial u_1}{\partial y_1} - \sigma_{xy}(x, y) = A_1(x).$$

As was already mentioned, a laminar boundary with a suction is two-layer and consists of the suction zone of thickness $O(\alpha)$ and the main part of thickness $O(\nu^{1/2})$. Substitution of $\beta = \nu$ and $\delta_2 = \nu^{1/2}$ into formula (6) makes it possible to formally rearrange it for the laminar case:

$$\mathrm{Re}_1 = \alpha^2\delta^{-3/2}. \tag{7}$$

The critical value of $\alpha = \alpha_* = \nu^{2/4}$ separates the regime of creeping motion [16] from the regime of ideal-liquid flow [17].

Zones 3 and 4. We revert to the problem on the turbulent-boundary-layer suction. In the buffer zone 3 of thickness δ_3, the velocity u_x is of the order of $O(1)$ and the viscous forces are negligibly small. Since $u_y \approx \beta v_3(x, y_3)$, from (2) we find that $\delta_3 = \beta$ and $u_x \approx u_3(x, y_3)$. From (1) it follows that $\varepsilon_3 = \beta$ and

$$u_3 \frac{\partial u_3}{\partial x} + v_3 \frac{\partial u_3}{\partial y_3} = -\frac{\partial \sigma_{xy}(x, y_3)}{\partial y_3}.$$

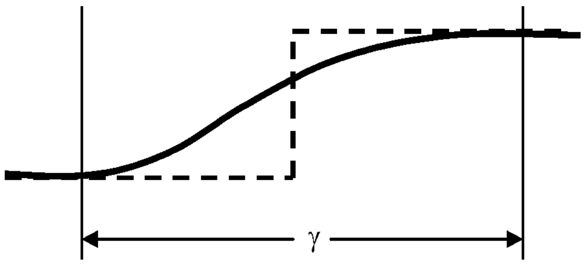
These estimates formally correlate with the data obtained in [4] for the boundary layer without suction. Analogous expansions are also valid in zone 4 of thickness δ_4:

$$u_y \approx \beta v_4(x, y_4), \quad u_x \approx 1 + \frac{\beta}{\delta_4} u_4(x, y_4).$$

From (1) we obtain that $\beta = \varepsilon_4$. Consequently, the turbulent stresses in zones 3 and 4 are equal in order of magnitude: $\varepsilon_4 = \varepsilon_3$. From this equation it follows that

$$\frac{\partial u_4}{\partial x} = -\frac{\partial \sigma_{xy}(x, y_4)}{\partial y_4}.$$

Conditions at the Corrsin Discontinuity. The next asymptotic layer is the Corrsin superlayer 5. A discontinuity cannot exist in the nature. All along, there is a small scale γ at which the discontinuity is structured, i.e., is smoothed (Fig. 2), because a time or space discontinuity can result only from the resimplification of the physical



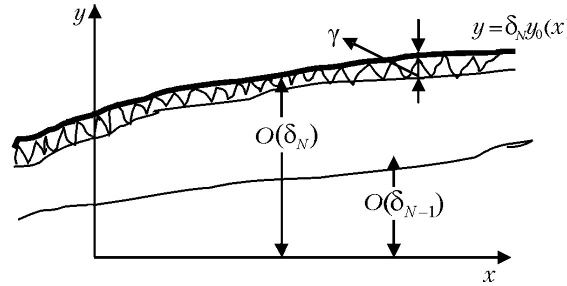


Fig. 2. Discontinuity (dashed line) and structure of the superlayer (solid line).

Fig. 3. Structure of the shell of the boundary layer (the superlayer is shaded, N = 4; the center line is the conditional boundary of the layer positioned below upstream of the superlayer).

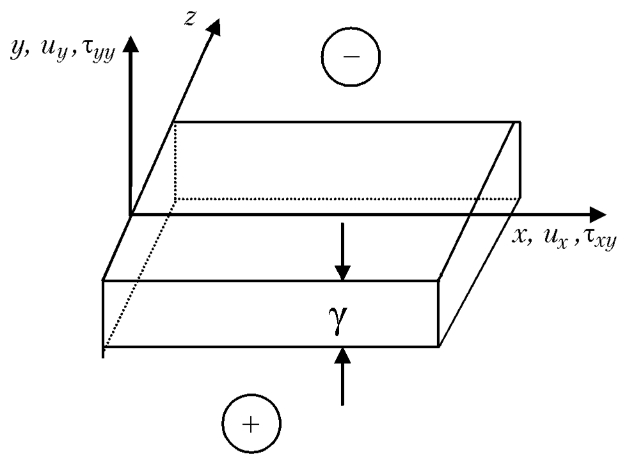


Fig. 4. Plane element of a discontinuity.

model or the idealization of the problem. Such a "jump–layer" dualism has the following form: a layer is realized in the scale $L = O(\gamma)$ and a jump in the scale $L >> \gamma$.

We will assume that the velocity components are nondisturbed and are defined by u_∞ and the pressure p is determined as ρu_∞^2. The main small parameters used in the formulation of the problem are the friction speed u_* and the reciprocal Reynolds number $\delta^2 = \nu/(lu_\infty)$.

Let us assume that the Corrsin *superlayer* is a discontinuity of turbulent stresses, and the pulsations at its outer boundary are zero. Since this layer is a superlayer (Fig. 3), it has certain features. Actually, the thicknesses δ_n of the usual layers enclosed into each other are related by the asymptotic recursion inequalities $\delta_{n+1} >> \delta_n$ (δ_n is a *sublayer* for δ_{n+1}). This situation is realized in the above-described cases of a free interaction, a boundary layer, and a turbulent wake. In the case being considered, in contrast, the thickness of the superlayer is smaller than the thickness of the previous layer: $\gamma = \delta_{N+1} << \delta_N$ ($N + 1$ is the number of the superlayer and δ_{N+1} is a superlayer relative to δ_N, N = 4).

In the scale $O(\delta_4)$, the superlayer is a discontinuity having a clearly defined boundary: $y = \delta_4 y_0(x) + O(\delta_4)$. At this boundary, the joining conditions following from the conservation laws are fulfilled.

Let us consider a locally plane element of a stationary jump x, y (the conditions along the z axis are identical to the conditions along the x axis) presented in Fig. 4. The quantities for the region above the discontinuity will be denoted by the upper index "–" and the quantities for the region under the discontinuity will be denoted by the upper index "+." From the condition of mass conservation we obtain

$$\overset{+}{u}_y = \overset{-}{u}_y = u_y \,.$$

Contrary to the usual practice [18], it will be assumed that, under the conditions of longitudinal-momentum conservation, a liquid flows through the layer and, consequently, a momentum is transferred through it:

$$\tau_{yy} + p^{+} - 2\delta^2 \frac{\partial u_y^{+}}{\partial y} = p^{-} - 2\delta^2 \frac{\partial u_y^{-}}{\partial y} , \quad (8)$$

$$\tau_{xy} + u_y u_x^{+} - \delta^2 \frac{\partial u_x^{+}}{\partial y} = u_y u_x^{-} - \delta^2 \frac{\partial u_x^{-}}{\partial y} , \quad (9)$$

where $\tau_{kj}^{-} \equiv 0$; $\tau_{kj}^{+} = \tau_{kj}$.

Structure of a Superlayer. In the scale $O(\gamma)$, a clearly defined front is absent and a transition layer, at both boundaries of which the join conditions are fulfilled, exists. The parabolic Reynolds equations, representing an approximation sufficient for the attainment of the further purposes, have the form

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + \frac{\partial p}{\partial x} = \delta^2 \frac{\partial^2 u_x}{\partial y^2} - \frac{\partial \tau_{xy}}{\partial y} , \quad (10)$$

$$u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + \frac{\partial p}{\partial y} = \delta^2 \frac{\partial^2 u_y}{\partial y^2} - \frac{\partial \tau_{yy}}{\partial y} , \quad \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 .$$

The superlayer is positioned in the velocity-defect zone, dividing the plus- and minus-regions. In the plus-region, according to the estimations of (4), we have

$$u_x = 1 + O(u_*) , \quad u_y = O(u_*^2) , \quad \delta_4 = u_* << 1 , \quad p = O(u_*^2) , \quad \tau_{xy} = O(\tau_{yy}) = O(u_*^2) .$$

The term responsible for the fluid flow under condition (9) is small as compared to the turbulent stress τ_{xy}. Actually:

$$u_y u_x^{+} - u_y u_x^{-} = u_y (u_x^{+} - u_x^{-}) = O(u_*^3) ,$$

whereas $\tau_{xy} = O(u_*^2)$. Consequently, instead of (9), we obtain

$$\tau_{xy} = \delta^2 \frac{\partial (u_x^{+} - u_x^{-})}{\partial x} .$$

The jump (to zero) of the turbulent stresses is due to the action of viscous forces. From these considerations we can easily determine the thickness of the superlayer. Since

$$\delta^2 \frac{\partial u_x}{\partial y} = O\left(\delta^2 \frac{u_*}{\delta_4}\right) = O(\tau_{xy}) = O(u_*^2) , \quad \text{then} \quad \delta_5 = \gamma = \frac{\delta^2}{u_*} = \delta_2 << \delta .$$

As expected, the thickness of the superlayer is equal to the thickness of the viscous near-wall layer since, in both cases, one and the same viscous and turbulent stresses act. The asymptotic expansions in the superlayer have the form

$$u_y = u_* \delta v(x, y_5) + \ldots , \quad u_x = 1 + u_* u_5(x, y_5) + \ldots , \quad -\infty \le y_5 \le +\infty ,$$

$$p = u_*^2 p_5 (x, y_5) \ldots , \quad \tau_{kj} = u_*^2 \sigma_{kj} (x, y_5) + \ldots , \quad y = \delta_4 y_0 (x) + \gamma y_5 .$$

From (10) we obtain

$$\frac{\partial u_5}{\partial y_5} = \sigma_{xy} , \quad p = - \sigma_{yy} , \quad \frac{\partial u_5}{\partial x} + \frac{\partial v_5}{\partial y} = 0 .$$

At $y_5 \to +\infty$ we properly have $\sigma_{xy} \to 0$, $\sigma_{yy} \to 0$, and, therefore, $p \to 0$. Consequently, the line of the turbulent-stress discontinuity is an isobar separating the plus-region from the minus-region. The flow in the minus-region is laminar and irrotational, $\delta_6 = \delta_4$.

Thus, two zones — the Corrsin superlayer and the irrotational velocity-defect zone — appear in addition to the four turbulent-boundary-layer zones detected earlier.

It should be noted that the theory devised in [4] is based on the supposition that the flow is plane and stationary. In actual practice, a turbulent boundary layer is three-dimensional and nonstationary.

Since the averaged-velocity component along the z axis $u_z(x, y)$ is equal to zero: $(\overline{u_z'}) = 0$, it follows from the Reynolds equations that the z component of the forces caused by the action of the turbulent tresses is also equal to zero

$$\frac{\partial \overline{u_x'' u_z''}}{\partial x} + \frac{\partial \overline{u_y'' u_z''}}{\partial y} + \frac{\partial \overline{u_z'' u_z''}}{\partial z} = 0 . \tag{11}$$

In the theory of (4) it is assumed that the turbulent-stress components are equal in order of magnitude. On the assumption that $u_x'' u_z'' \sim u_y'' u_z'' \sim u_z'' u_z''$, from (11) we obtain that $u_y'' u_z'' = f(x)$ in the boundary layer where $x, z \sim l$ and $y << l$. This equality is true not only in the boundary layer, but also outside it. Since the turbulence in the incident flow is uniform, $f = \text{const}$.

The second comment concerns the time dependence of the solution. To estimate this dependence, it is necessary to change the rule of averaging — to perform the *averaging over phase* [18] instead of the *time averaging* proposed by Reynolds [9]. Then the term $\partial u_*/\partial t$ responsible for the nonstationarity will appear in the Reynolds equation that, in this case, will become suitable for description of wave process. In the viscous near-wall zone, the order of this term is u_*/T, and the order of the friction force is u_*^3/δ^2. On equating these two estimates, we obtain the characteristic period of a wave: $T \sim (\delta/u_*)^2 << 1$. Thus, high-frequency waves propagate in the near-wall zone. In the buffer zone, $u_x \sim 1$; therefore, $T \sim 1$. In the velocity-defect zone, the velocity component u_x is linearized and, once again, $T \sim 1$.

Conclusions. The asymptotic structure of a turbulent boundary layer of an incompressible fluid with a suction, formed on a plate, has been investigated. In this case, the combined method of different scales and joined asymptotic expansions was used. Unlike the laminar boundary layer joined asymptotically with the external flow, the turbulent boundary layer has a clearly defined boundary representing the Corrsin superlayer. Unlike the Ponomarev theory, we took into account the existence of this boundary and the influence of the suction on the structure of the flow and determined the conditions, under which the viscous and nonviscous regimes of flow are realized in the suction zone. Moreover, the estimates of the quantities characteristic of the other asymptotic regions were compared to those obtained in [4].

The Corrsin discontinuity is unusual. Discontinuities of two types are known [19]: the discontinuity of the normal velocity component caused by the compressibility of the fluid and the discontinuity of the tangential velocity component, through which a flow is absent. The boundary between the laminar and turbulent regions in the incompressible fluid cannot be assigned to any of these types because the normal velocity component does not undergo a discontinuity at it and there is a flow through this boundary.

The structure of a *hydrodynamic discontinuity* of a continuous medium, where $\gamma >> \lambda$, was considered; an example is a tangential discontinuity: $\gamma = \delta >> \lambda$. The structure of a *nonhydrodynamic discontinuity* can be introduced within the framework of the hydrodynamic approximation. In this case, it is necessary to appeal to the kinetic theory; an example is a strong shock wave in a gas: $\gamma \sim \lambda$. In accordance with the results obtained, the Corrsin superlayer represents a hydrodynamic discontinuity. Within this layer, as in the near-wall zone, the molecular-viscosity action is significant. The thickness of the superlayer was found to be equal in order of magnitude to the thickness of the viscous neat-wall layer. Since this value is fairly small from the practical standpoint, the experimental data on the structure of the Corrsin superlayer are small in numbers.

The use of asymptotic methods for solving the problem on turbulent fluid flows is complicated since the form of the Reynolds stresses is unknown. Therefore, the conclusions made in this case are not as rigorous as the conclusions made for the boundary layer without suction.

In the problems on a jet and a mixing layer, such parameters as the wall and dynamic velocity are absent; therefore, at fairly large distances from the "flow origin" the asymptotic structure of the layers will be three-zone, including a main zone, a superlayer with a structure identical to the structure considered, and a laminar-flow zone.

The asymptotic analysis, along with the phenomenological analysis [20], gives a fairly strict qualitative representation of the phenomenon being considered, without which it is impossible to obtain quantitative results with the use of the zonal numerical methods [21] and the experimental diagnostics, and, therefore, opens up new possibilities. For numerical simulation, such analysis gives a recipe for choosing an economical grid and elimination of unnecessary terms in the Navier–Stokes equation for certain zones. In an experiment, the situation is more complex. Since the thickness of each sublayer is smaller (by approximately ten times) than the thickness of the previous sublayer by an order of magnitude, measuring devices should be positioned in the near-wall zone with an accuracy higher by several orders of magnitude than the accuray of disposition of devices in the other zones. Because of this, at present there are no reliable data on the structure of these sublayers.

NOTATION

$A_1(x)$, $A_2(x)$, arbitrary functions; p, pressure; Re, Reynolds number; T, period of oscillations; u, v, velocity components; u_∞, wake velocity; u_*, friction speed; x, y, Cartesian coordinates; α, period of slots; β, suction coefficient; β_*, critical value of the suction coefficient; γ, length scale; δ, thickness of a layer; λ, mean free path; ν, coefficient of kinematic viscosity; ρ, density of a fluid; τ_{xy}, Reynolds stresses.

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